

Detecting chromosomal aberrations using Hidden Markov Models with inhomogenous Markov chains

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Background

Chromosomal Aberrations

- Change in chromosomal structure or number of chromosomes
- Effects:
 - Physical or mental abnormalities
 - Developmental problems
 - Cancer as an effect of accumulated aberrations

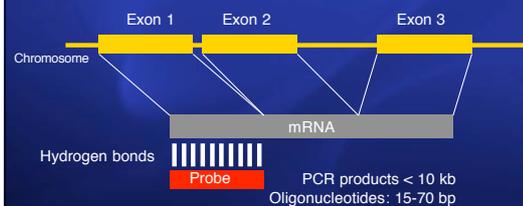
Copy numbers

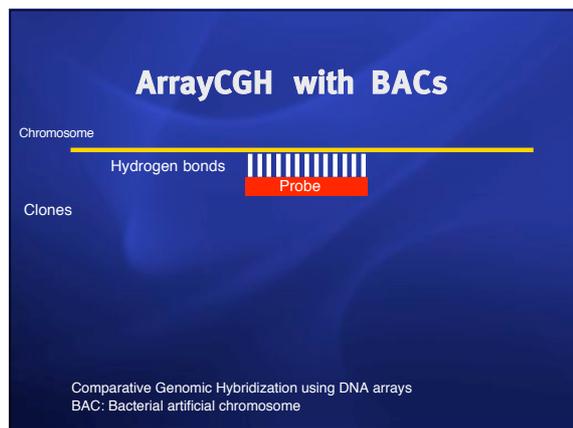
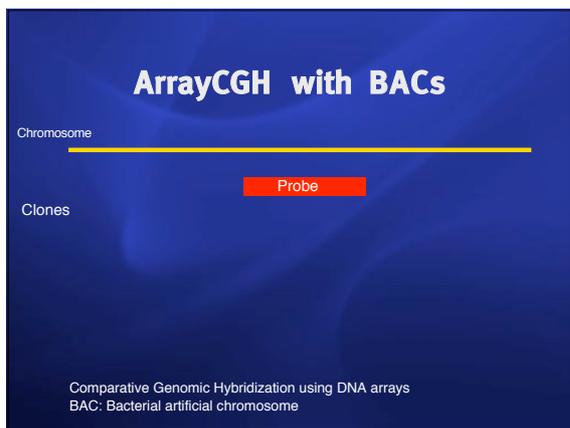
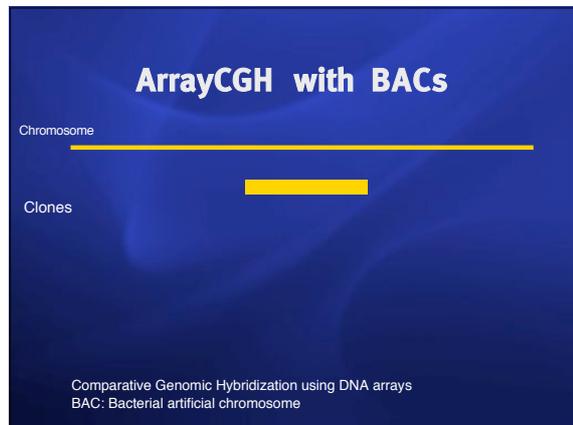
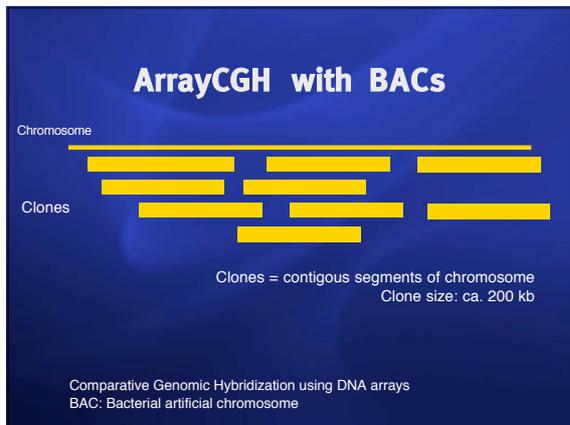
- Copy number is *usually* 2 in diploid genomes
 - Neglecting copy number polymorphisms*
- Aberrations = copy number changes

Detection

- Compare cohorts: Test vs Reference
- Idea “*differential gene expression*”
 - Use DNA-Microarrays
 - Probes cover chromosomes
 - Hybridize DNA (not mRNA) to chips

Gene expression





Copy numbers and hybridization strength

- Comparing hybridization of test versus reference under *identical conditions*
 - Less hybridization \leftrightarrow segment lost
 - No change
 - More hybridization \leftrightarrow segment gained

Warning: Oversimplification

Goal

Goal

Segment the chromosome into regions

- Unchanged
- Lost
- Gained

Observations

Standard Gene Expression Analysis

Assumptions

- most expression levels are unchanged
- Independence between genes (loci)

Analysis

- Determine background, normalize expression values
- Compute p-values corrected for multiple testing

ArrayCGH

Data quality not sufficient for deciding *per position*

- Change of expression not significant
- Frequent errors

(see: J. Toedling, S. Schmeier, M. Heinig, B. Georgi and S. Röpcke. MACAT - MicroArray Chromosome Analysis Tool. Bioinformatics. 2004.)

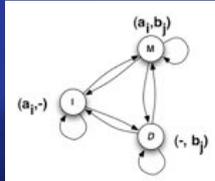
Proximity effect

- The larger the distance between neighboring probes, the more likely a breakpoint can occur
- Gains and losses affect regions

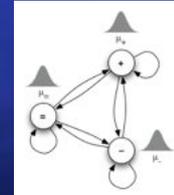
Method

Following Jane Frydland (UCSF) et. al.

Idea: Sequence alignment



Loss/Gain-Model



$$\mu_- < \mu_0 < \mu_+$$

Hidden Markov Models

- Markov chain over hidden states
 - transition matrix $A = \{ a_{ij} \}$
 - $a_{ij} = P[\text{state } j \mid \text{state } i]$
 - first order, time-homogenous
- Continuous emissions:
 - Density function per state: gaussian $N(\mu, \sigma)$, mixture of Gaussians
 - Emission only depends on state producing it

Hidden Markov Model

- Copy number \Leftrightarrow state
- Produces sequence of hybridization intensities x_1, x_2, \dots, x_n
- Viterbi path (most likely state sequence) yields segmentation sequence

Caveat: copy numbers ratios

- +/- = too simple
- Mixture estimation/clustering using AIC/BIC
- Mixture yields:
 - Number of distinct copy numbers
 - Relative hybridization per copy number

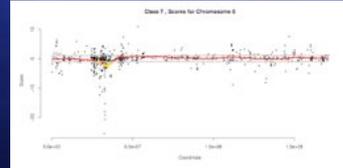
Method Extension

State mixture densities

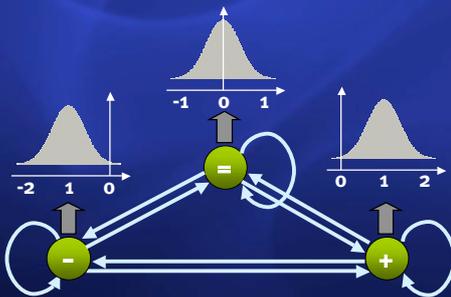
- Relative hybridization intensities are not normal
- Observation for ArrayCGH: Many components mean close to zero
- "="-state: collect components with similar means. Use mixture density.

Measurement errors

- About 5% of x_i are erroneous
 - Segments are fragmented
 - Too many segments are missed



Hidden Markov Model



Extended Model: Error component



Proximity

- So far: Equal distance assumption
 - Segment length measured in *number of probes*
- How to incorporate distance in base pairs?

ArrayCGH



Overlapping clones should obviously be correlated

ArrayCGH



Distance correlations

Goal:

- Probes should have a higher probability of the same annotation (=,+,-) when they overlap
- The self-transition probability of states should increase as the overlap increases

Time-Inhomogenous Markov Chains

- Usual Markov assumption:
 $P[q_t = j | q_{t-1} = i, q_{t-2} = \dots] = P[q_t = j | q_{t-1} = i]$
 Transition matrix A
- Time-Inhomogeneity
 $P[q_t = j | q_{t-1} = i]$ depends on parameter t
 Transition matrix $A(t)$
- Example: Simulated Annealing

Inhomogenous HMM

HMM with a inhomogenous Markov chain t can be

- Time (sequence index)
- Depend on sequence values (sum of observations)
- Usual algorithms still work

B. Knab, A. Schilp, B. Steckmetz, B. Wichem, Model-based clustering with Hidden Markov Models and its application to financial times series data, (GSI 2002)

Transition classes

How to model dependence on overlap?



Transition classes

$$a_{=+} = \begin{matrix} p_1, & \text{if overlap} = 0\% \\ \dots \\ p_k, & \text{if overlap} > 90\% \end{matrix}$$

- Computationally efficient
- Simple to implement
- Reasonable approximation

Technicalities

- Initial parameter estimates
- Missing values: $Ru\{M\}$
- Coupling transition matrices in training:
 - avoid overfitting
 - fixed ratios of state duration between matrices

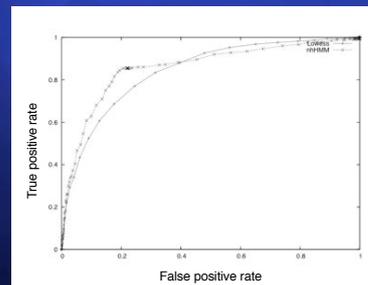
Michael Selfert: Analyzing Microarray Data Using Homogenous and Inhomogenous Hidden Markov Models. Diplom LMU Halle.

Discussion

Results

- Consistency tests successful
- Robust: missing data as well as noise
- Collaboration on ArrayCGH with Dept. Ropers
 - Expert evaluation in comparison with prior work. Working on publication
 - Caveat: No large-scale cross-validation results

Lai et al. simulated data (no BACs)



Outlook: Method

- Using test statistic per clone instead of expression values
- Theory: Learning non-homogenous structure

Perspective

Two interesting sources of data

- ArrayCGH
 - Relatively scarce data (in-house Dept. Ropers)
- Gene expression:
 - Ample, freely available, low quality
 - Case study: Pollak et al. PNAS (2002), Breast cancer data, Chr. 17

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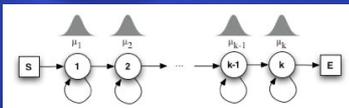
Perspective

- Combination of ArrayCGH and gene expression:
 - Regulation in presence of aberrations

k-Segmentation (Picard)

Find k Normals and $k-1$ segment boundaries maximizing the obvious likelihood function for X_1, X_2, \dots, X_n

k-Segmentation (Picard)



- All transition probabilities are $1/2$
- Assuming homogenous variance σ
- Can show: Maximum likelihood HMM and Viterbi-path converge to solution of k-Segmentation problem when σ goes to zero

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- Jane Frydland for helpful discussions

Thanks

<http://ghmm.org>

<http://algorithmics.molgen.mpg.de>